Girraween High School

Year 12 Half Yearly Examination 2017

Mathematics Extension 2

Time Allowed-Two hours (Plus 5 minutes reading time)

Instructions:

*Attempt all questions.

*Show all necessary working.

*Marks may be deducted for careless or badly arranged work.

*Board – approved calculators may be used

1. The polynomial $P(x)$ with real coefficients has $x=1$ as a root of multiplicity 2 and $x+i$ as a factor. Which one of the following expressions could be a factorised form of $P(x)$?

A) $(x^2+1)(x-1)^2$ B) $(x+i)^2(x-1)^2$ C) $(x-i)^2(x-1)^2$ D) $(x^2+1)(x-i)^2$

2. What are the foci of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$?

A) ($(0,\pm3)$ B) $(\pm5,0)$ C) $(\pm3,0)$ D) $(0,\pm5)$

3. Given that $z=1+i$, what is the value of z^8?

A) -16 B) -8 C) 8 D) 16

4. The equation $x^4+px+q=0$, where $p\neq0$ and $q\neq0$, has roots α,β,γ and δ. What is the value of $\alpha^4+\beta^4+\gamma^4+\delta^4$?

A) $-4q$ B) p^2-2q C) p^4-2q D) p^4

5. The sum of the eccentricities of two different conics is $\frac{3}{4}$.

Which pair of conics could this be?

A) Circle and ellipse B) Ellipse and parabola C) Parabola and hyperbola D) Hyperbola and circle

Question 6. (21 marks)

a) If $z = 1+2i$, find in the form $a+ib$, where a and b are real, the values of

i) $iz + \overline{z}$ 1

ii) $\dfrac{1}{z}$ 1

b) The points A and C represent the complex numbers $1+i$ and $7+3i$ respectively. Find the complex number ω, represented by B such that ΔABC is isosceles and right angled at B. 3

c) Shade on an Argand diagram the region represented by the complex number z where $\dfrac{\pi}{4} \le \arg z \le \pi$, $1 \le \mathrm{Im}(z) \le 3$ and $|z| \le 4$ 3

d) Let $\alpha = 1 + i\sqrt{3}$ and $\beta = 1+i$

i) Find $\dfrac{\alpha}{\beta}$, in the form $x+iy$. 1

ii) Express α in modulus-argument form. 1

iii) Given that β has the modulus-argument form $\beta = \sqrt{2}\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$,

find the modulus-argument form of $\dfrac{\alpha}{\beta}$. 2

iv) Hence find the exact value of $\sin\dfrac{\pi}{12}$. 1

e) i) Using De Moivre's theorem, find an expression for $\cos 3\theta$ in terms of $\cos\theta$.
3

ii) If $\cos 3\theta = -\dfrac{1}{\sqrt{2}}$, show that $x = 2\sqrt{2}\cos\theta$ satisfies the equation $x^3 - 6x + 4 = 0$. 2

iii) Hence, find the exact values of the roots of this equation. 3

Question 7. (18 marks)

a) The cubic polynomial $x^3+2x^2-5x-6=0$ has roots α, β and γ. Find the values of

i) $\alpha+\beta+\gamma$ **1**

ii) $\alpha^2+\beta^2+\gamma^2$ **2**

iii) $\alpha^3+\beta^3+\gamma^3$ **2**

b) If $2+i$ is a root of $P(x)=x^4-6x^3+9x^2+6x-20$, resolve $P(x)$ into linear factors over the field of complex numbers. **4**

c) The equation $x^3+2x+1=0$ has roots α, β and γ. Find in expanded form the

monic cubic equation with roots $\alpha+1, \beta+1$ and $\gamma+1$. **2**

d) i) Prove that the condition for the roots of $x^3-px^2+qx-r=0$ to be in

arithmetic progression is $2p^3-9pq+27r=0$. **4**

ii) Hence solve the equation $x^3-12x^2+39x-28=0$ **3**

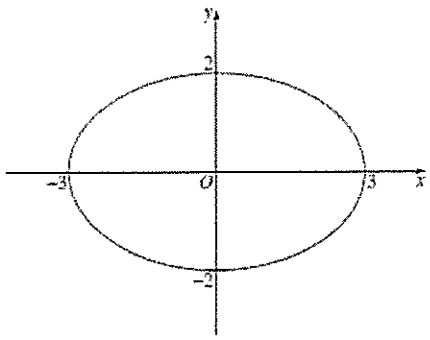
e) i) If x^3+px+q has a double zero, prove that the double zero must be $-\dfrac{3q}{2p}$, $p \neq 0$ **2**

ii) Show also that if x^3+px+q has a double zero, then $4p^3+27q^2=0$ **2**

Question 8 (15 marks)

a) Find the equation of the tangent to the curve $x^2 - xy + y^3 = 1$ at the point $P(1,1)$ on the curve. **3**

b) The diagram below shows an ellipse.



i) Write an equation for the ellipse. **1**

ii) Find the eccentricity of the ellipse. **1**

iii) Write the coordinates of the foci of the ellipse. **1**

iv) Write the equations of the directrices of the ellipse. **1**

c) i) Show that the equations of the tangent and the normal at $P(x_1, y_1)$ to the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

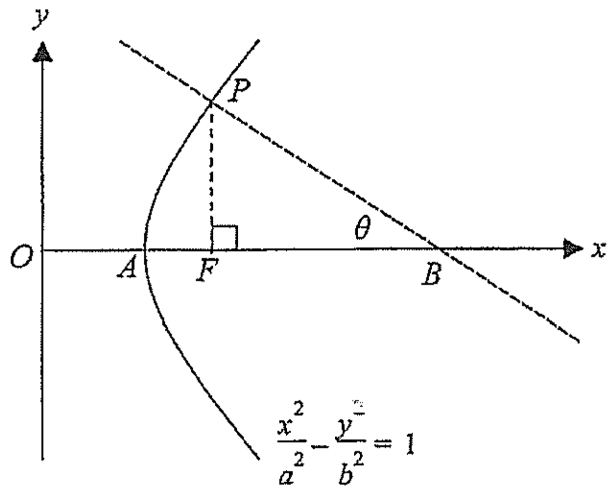
are $\dfrac{xx_1}{a^2}+\dfrac{yy_1}{b^2}=1$ and $\dfrac{a^2x}{x_1}-\dfrac{b^2y}{y_1}=a^2-b^2$ respectively. **4**

ii) The tangent at P meets the x-axis at T. PN is perpendicular to the x-axis, and the normal at P meets the x-axis at G. Show that $OT \times NG = b^2$, where O is the centre of the ellipse. **4**

Question 9 (10 marks)

a) Sketch the hyperbola with equation $\frac{x^2}{64}-\frac{y^2}{36}=1$, showing the foci, the directrices and the asymptotes. **4**

b)



In the diagram, F is a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with eccentricity e.

This branch of the hyperbola cuts the x-axis at A where $AF=h$. P is the point on the hyperbola vertically above F and the normal at P cuts the x-axis at B making an acute angle θ with the x-axis. (Assume that the equation of the normal at ($P(x_1,y_1)$ is given by

$\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$. **Do not prove this)**

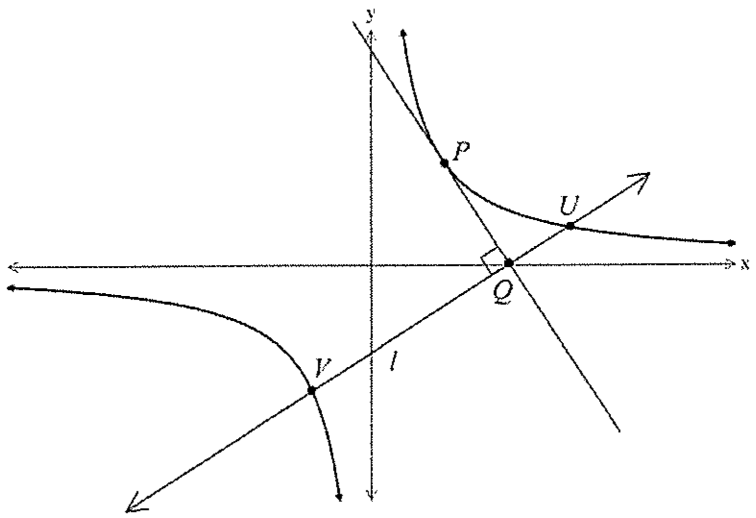
i) Show that $\tan\theta=\frac{1}{e}$ **3**

ii) Show that $PF=h(e+1)$ **3**

Question 10. (9 marks)

A tangent is drawn at any point $P\left(ct,\frac{c}{t}\right)$ on the hyperbola $xy=c^2$. This tangent meets the x axis at Q. Through Q a straight line l is drawn perpendicular to the tangent.

The line l cuts the hyperbola in the two points U and V.



i) Show that the equation of the tangent is $x+t^2y=2ct$ **2**

ii) Find the coordinates of Q **1**

iii) Find the equation of line l. **1**

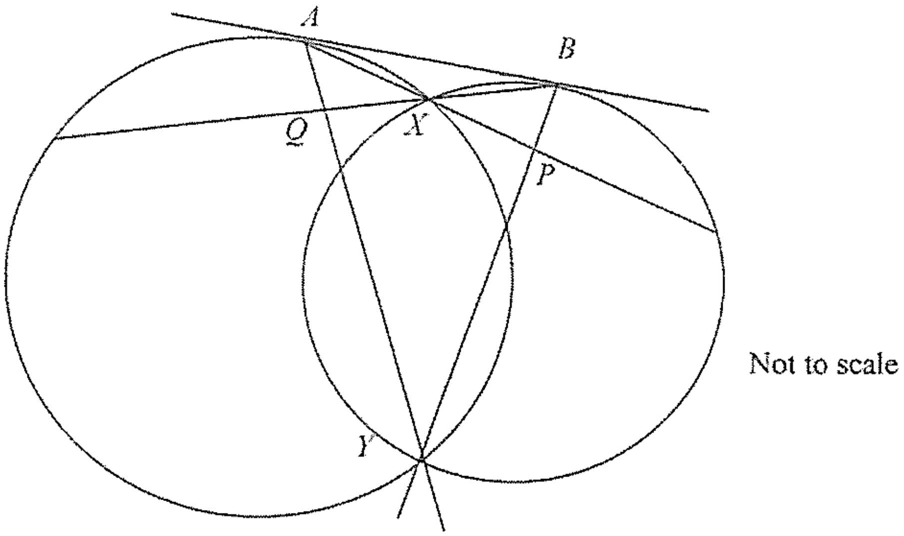
iv) If M is the midpoint of the interval UV, show that the coordinates of M are $\left(ct,-ct^3\right)$. **3**

v) Hence find the locus of M as the point P varies. **2**

Question 11 (7 marks).

Two circles intersect at X and Y and AB is a common tangent.

The lines AX and BY meet at P. The lines AY and BX meet at Q as shown in the diagram below.



i) Copy the diagram in your answer booklet and show that $XPYQ$ is a cyclic quadrilateral, giving reasons. **4**

ii) Prove that $QP \parallel AB$. **3**

END OF EXAMINATION

Year 12 Mathematics Ext. 2 2017

Task 2 SOLUTIONS

Multiple choice Questions

① $(x+i)$ is a factor. $\therefore$ $(x-i)$ is also a factor

$(x+i)(x-i) = x^2+1$

and root of multiplicity 2 $\Rightarrow (x-1)^2$

$\therefore$ (A)

② $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1 \Rightarrow a=3$ and $b=4$.

$b^2 = a^2(e^2-1)$

$16 = 9(e^2-1)$

$\therefore e = \dfrac{5}{3}$

(B) $\therefore$ Foci $= (\pm 5, 0)$

③ $1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$(1+i)^8 = 16(\cos 2\pi + i\sin 2\pi) = 16$

(D)

④ $x^4+px+q=0$ and $\alpha, \beta, \gamma, \delta$ are roots.

$\therefore \alpha+\beta+\gamma+\delta = 0$

$\therefore \alpha^4 = -p\alpha - q$

$\beta^4 = -p\beta - q$

$\gamma^4 = -p\gamma - q$

$\delta^4 = -p\delta - q$

$\alpha^4+\beta^4+\gamma^4+\delta^4 = -p(\alpha+\beta+\gamma+\delta) - 4q$

$= -4q$

(A)

⑤ Circle has $e=0$, ellipse has $e<0$.

parabola has $e=1$, hyperbola has $e>1$

$\therefore$ (A)

Question 6

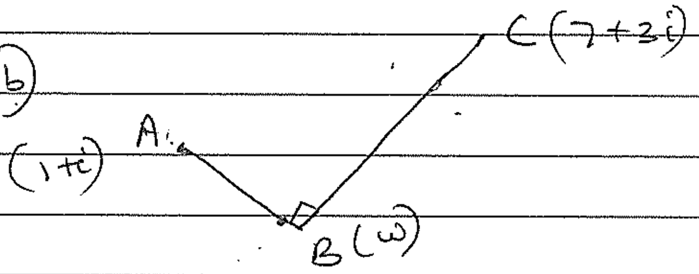
a) (i) $z = 1+2i$

$\therefore iz + \bar{z} = i(1+2i) + 1 - 2i$

$= i - 2 + 1 - 2i = -1 - i$ (1)

(ii) $\dfrac{1}{z} = \dfrac{\bar{z}}{z\bar{z}} = \dfrac{\bar{z}}{|z|^2} = \dfrac{1-2i}{1^2+2^2} = \dfrac{1}{5} + \left(\dfrac{-2}{5}\right)i$ (1)

b)



$\overrightarrow{BA} = \overrightarrow{BC}\, i$

$1+i-\omega = (7+3i-\omega)i$

$1+i-\omega = 7i - 3 - \omega i$

$\omega(1-i) = 1+i-7i+3 = 4-6i$

$\therefore \omega = \dfrac{4-6i}{1-i} = \dfrac{4-6i}{1-i} \times \dfrac{1+i}{1+i}$

$= \dfrac{4+6-2i}{2} = \underline{\underline{5-i}}$

OR

$\overrightarrow{BC} = i\,\overrightarrow{BA}$

$(7+3i) - \omega = i(1+i-$

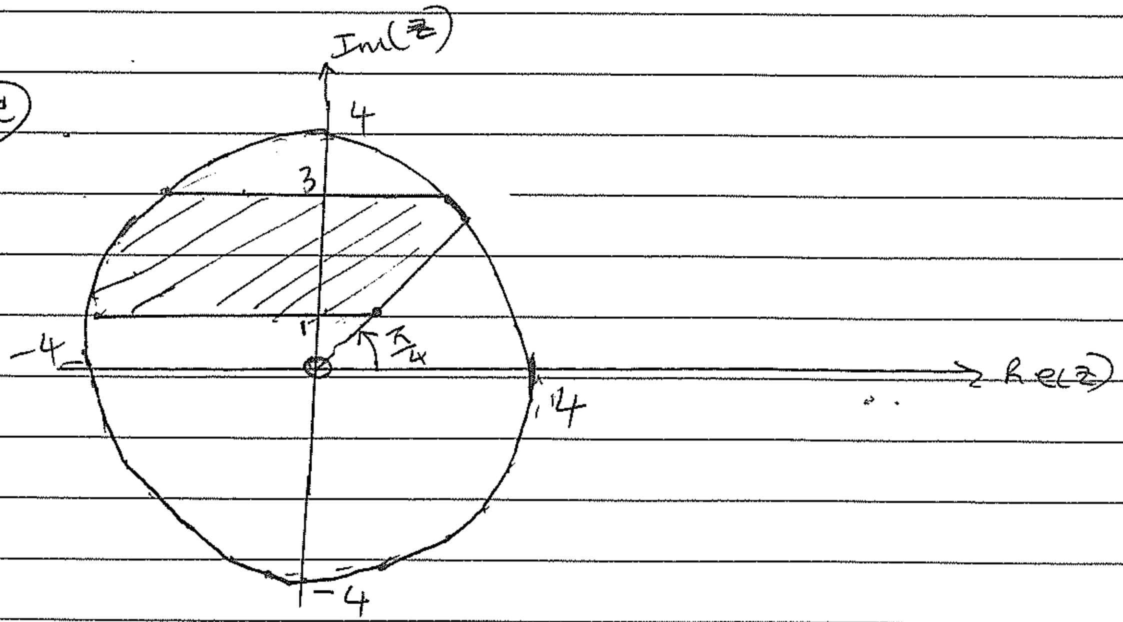
$7+3i-\omega = i-1-i\omega$

$\omega(i-1) = -2i-8$

$\therefore \omega = \underline{\underline{3+5i}}$

(3)

c)



d) $\alpha = 1+\sqrt{3}i$

$\beta = 1+i$

(i) $\therefore \frac{\alpha}{\beta} = \frac{1+\sqrt{3}i}{1+i} = \frac{1+\sqrt{3}i}{1+i} \times \frac{1-i}{1-i}$

$$= \frac{1+\sqrt{3} + i(\sqrt{3}-1)}{2} \quad ①$$

(ii) $\alpha = 1+\sqrt{3}i = 2\operatorname{cis}\frac{\pi}{3}$ ①

~~Given~~

(iii) $\beta = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$,

$$\frac{\alpha}{\beta} = \frac{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) \quad ②$$

(iv) $\frac{\alpha}{\beta} = \frac{1+\sqrt{3}}{2} + i\frac{(\sqrt{3}-1)}{2} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

equating the imaginary parts;

$$\frac{\sqrt{3}-1}{2} = +\sqrt{2}\sin\frac{\pi}{12}$$

$$\therefore \sin\frac{\pi}{12} = +\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \frac{-1+\sqrt{3}}{2\sqrt{2}} \quad ①$$

d) (i) $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$, using De moivre's Theorem

Let $c = \cos\theta$, $s = \sin\theta$

LHS $\Rightarrow c^3 + 3c^2(is) + 3c(is)^2 + (is)^3$

$$= (c^3 - 3cs^2) + i(3c^2s - s^3)$$

equating real parts $\Rightarrow c^3 - 3cs^2 = \cancel{\cos^3\theta - 3}$

$$= \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$$

$$\therefore \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

Question 6 continued.

(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = \frac{-1}{\sqrt{2}}$

substitute $\cos\theta = \frac{x}{2\sqrt{2}}$ gives,

$$\frac{4x^3}{16\sqrt{2}} - \frac{3x}{2\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$\therefore 4x^3 - 24x = -16 \Rightarrow x^3 - 6x + 4 = 0.$

$\therefore x = 2\sqrt{2}\cos\theta$ satisfies the equation $x^3 - 6x + 4 = 0.$

(iii) $\cos 3\theta = -\frac{1}{\sqrt{2}} \Rightarrow 3\theta = \pm\frac{3\pi}{4} + 2k\pi, \; k = 0, 1$

$\therefore \theta = \frac{(8k \pm 3)\pi}{12}, \; k = 0, 1$

$= \frac{\pi}{4}, \frac{5\pi}{12}, \frac{11\pi}{12}.$

Question 7

$x^3+2x^2-5x-6=0$ has roots α, β and γ.

a) (i) $\alpha+\beta+\gamma = \boxed{-2}$ 1

(ii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= (-2)^2 - 2(-5) = 14$ 2

(iii) $\alpha^3+\beta^3+\gamma^3 = ?$

α is a root $\Rightarrow$ $\alpha^3 = -2\alpha^2+5\alpha+6$

β " " $\Rightarrow$ $\beta^3 = -2\beta^2+5\beta+6$

γ " " $\Rightarrow$ $\gamma^3 = -2\gamma^2+5\gamma+6$

$\alpha^3+\beta^3+\gamma^3 = -(\alpha^2+\beta^2+\gamma^2)+5(\alpha+\beta+\gamma)+18$

$= (-2\times 14)+5(-2)+18$

$= -28 -10 +18 = -20$ 2

b) $P(x) = x^4-6x^3+9x^2+6x-20$

If $(2+i)$ is a root, then $(2-i)$ is also a root

($\because$ $P(x)$ has real coefficients).

Let $z = 2+i$ & $\bar{z} = 2-i$

$z+\bar{z} = 4$ and $z\bar{z} = 5$

$\therefore$ The quadratic equation with roots $(2+i), (2-i)$ is

x^2-4x+5

Divide $P(x)$ by x^2-4x+5 to find other roots

$$
\begin{array}{r|l}
 & x^2-2x-4 \\
\hline
x^2-4x+5 & x^4-6x^3+9x^2+6x-20 \\
 & x^4-4x^3+5x^2 \\
\hline
 & -2x^3+4x^2+6x \\
 & -2x^3+8x^2-10x \\
\hline
 & -4x^2+16x-20 \\
 & -4x^2+16x-20 \\
\hline
\end{array}
$$

$\therefore P(x) = (x^2-4x+5)(x^2-2x-4)$

$= [x-(2+i)][x-(2-i)]$

$(x-1-\sqrt{5})(x-1+\sqrt{5})$

4

c) $x^3+2x+1=0$. has roots α, β and σ.

Let $y = x+1$ $\therefore$ $x = y-1$.

sub. in $x=y-1$ into $x^3+2x+1=0$ gives.

$(y-1)^3+2(y-1)+1=0$.

$\therefore y^3-3y^2+5y-2=0$

$\therefore x^3-3x^2+5x-2=0$.

2

d) (i) $x^3-px^2+qx-r=0$ — ①

Let the roots be $a-d, a, a+d$

$\therefore$ sum of roots $\Rightarrow$ $3a = p$

$\therefore a = \frac{p}{3}$

sub. in $\frac{p}{3}$ into ① as a is a roots of ①

4

$$\Rightarrow \frac{p^3}{27} - p\cdot\frac{p^2}{9} + q\cdot\frac{p}{3} - r = 0$$

$$p^3 - 3p^3 + 9pq - 27r = 0$$

$$\therefore 2p^3 - 9pq + 27r = 0.$$

(ii) solve $x^3-12x^2+39x-28=0$. — ②

equating co-efficients of ① and ② $\Rightarrow$ $p=12$ $\therefore a = \frac{12}{3} = 4$

$\therefore (x-4)$ is a root of ②

B1

Question 7 continued

(ii)
$$
\begin{array}{r}
x^2 - 8x + 7 \\
x-4 \,\big)\, \overline{x^3 - 12x^2 + 39x - 28} \\
x^3 - 4x^2 \\
\hline
-8x^2 + 39x \\
-8x^2 + 32x \\
\hline
7x - 28
\end{array}
$$

$\therefore \quad x^3 - 12x^2 + 39x - 28 = (x-4)(x-7)(x-1)$

4

e) Let $f(x) = x^3 + px + q$. — (1)

$f'(x) = 3x^2 + p$

double root $\Rightarrow$ $f'(x) = 0 \Rightarrow x^2 = \frac{-p}{3}$

sub. in $x^2 = \frac{-p}{3}$ into $f(x) = 0$.

$\therefore \quad x \cdot \left(-\frac{p}{3}\right) + px + q = 0$.

$\therefore \quad \frac{-px}{3} = +q$

$\therefore \quad x = -\frac{3q}{2p}$

2

This value of $x = -\frac{3q}{2p}$ satisfies both $f(x) = 0$ and $f'(x) = 0$ (double root)

$\therefore$ sub. in $x = -\frac{3q}{2p}$ into $f'(x) = 0$.

$3 \cdot \left(-\frac{3q}{2p}\right)^2 + p = 0 \rightarrow \frac{27q^2}{4p^2} + p = 0$

$\therefore \quad 27q^2 + 4p^3 = 0$.

2

Question 8

a) $x^2 - xy + y^3 = 1$.

Differentiating with respect to x gives,

$2x - xy' - y + 3y^2 \cdot y' = 0$

$2x - y + y'(3y^2 - x) = 0$

$\therefore y' = \dfrac{y - 2x}{3y^2 - x}$

at $(1,1)$, $y' = \dfrac{1-2}{3-1} = \dfrac{-1}{2}$

$\therefore$ equation: $y - 1 = \frac{-1}{2}(x - 1)$

$2y - 2 = -x + 1 \Rightarrow x + 2y - 3 = 0$

3

b) (i) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ $(a = 3, b = 2)$

(ii) $b^2 = a^2(1 - e^2)$

$\therefore 4 = 9(1 - e^2) \Rightarrow e^2 = 1 - \frac{4}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$

(iii) Foci $= (\pm ae, 0) = (\pm\sqrt{5}, 0)$

(iv) d: $x = \pm \dfrac{a}{e} \Rightarrow x = \pm \dfrac{9}{\sqrt{5}}$

Question 8 (continued)

c) (i) Differentiating both sides of

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}y'=0$$

$$\therefore y' = \frac{-b^2x}{a^2y}$$

at $P(x_1, y_1)$, the gradient is $= \frac{-b^2x_1}{a^2y_1}$

$\therefore$ The equation of tangent is.

$$y-y_1 = \frac{-b^2x_1}{a^2y_1}(x-x_1)$$

$$\therefore a^2yy_1 - a^2y_1^2 = -b^2xx_1 + b^2x_1^2$$

$$\therefore b^2xx_1 + a^2yy_1 = b^2x_1^2 + a^2y_1^2$$

$(\div a^2b^2)$ $\frac{xx_1}{a^2}+\frac{yy_1}{b^2} = \frac{x_1^2}{a^2}+\frac{y_1^2}{b^2} = 1$ (as (x_1, y_1) lie on the ellips

$$\therefore \frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$ 2

(ii) The equation of the normal:

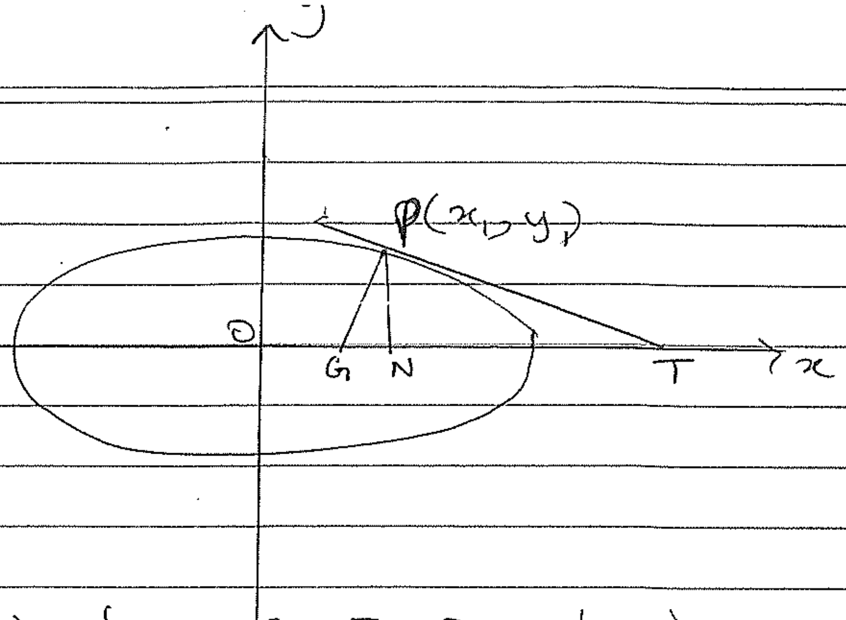
$$y-y_1 = \frac{a^2y_1}{b^2x_1}(x-x_1).$$

$$\therefore b^2x_1y - b^2x_1y_1 = a^2xy_1 - a^2x_1y_1$$

$$a^2y_1x - b^2x_1y = x_1y_1(a^2-b^2)$$

$(\div x_1y_1)$ $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2-b^2$ 2

(ii)



coordinates of T $\Rightarrow$ sub. in $y=0$

$$\therefore \frac{x x_1}{a^2} + 0 = 1 \Rightarrow x = \frac{a^2}{x_1}$$

$$\therefore OT = \frac{a^2}{x_1}$$

coordinates of G. $\Rightarrow$ sub - in $y=0$

$$\therefore \frac{a^2 x}{x_1} = a^2 - b^2 \Rightarrow x = \frac{x_1(a^2-b^2)}{a^2}$$

$$\therefore NG = x_1 - OG$$

$$= x_1 - \frac{x_1(a^2-b^2)}{a^2} = \frac{x_1 b^2}{a^2}$$

$$\therefore OT \times NG = \frac{a^2}{x_1} \times \frac{x_1 b^2}{a^2} = b^2$$

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow \quad y_1^2 = \frac{b^2}{a^2} \cdot b^2$$

$$\therefore y_1 = \frac{b^2}{a}$$

$\therefore$ the gradient of normal at P is

$$= -\frac{a \cdot \frac{b^2}{a}}{b^2 e} = \frac{-b^2}{b^2 e} = \frac{-1}{e}$$

$$\therefore \tan\theta = \frac{1}{e}$$

(ii) The equation of normal at P:

$$\frac{a^2 x}{x_1} + \frac{b^2 y}{y_1} = a^2 + b^2$$

$\therefore$ coordinates of $B \Rightarrow$ sub in $y = 0$.

$$\frac{a^2 x}{ae} = a^2 + b^2 \Rightarrow x = \frac{(a^2+b^2)e}{a}$$

$$\therefore FB = OB - OF = \left(\frac{a^2+b^2}{a}\right)e - ae$$

$$= \frac{a^2e + b^2e - a^2e}{a} = \frac{b^2 e}{a}$$

$$\text{Tan}\,\theta = \frac{PF}{FB} = \frac{PF}{\frac{b^2e}{a}} = \frac{1}{e} \quad (\text{from (i)})$$

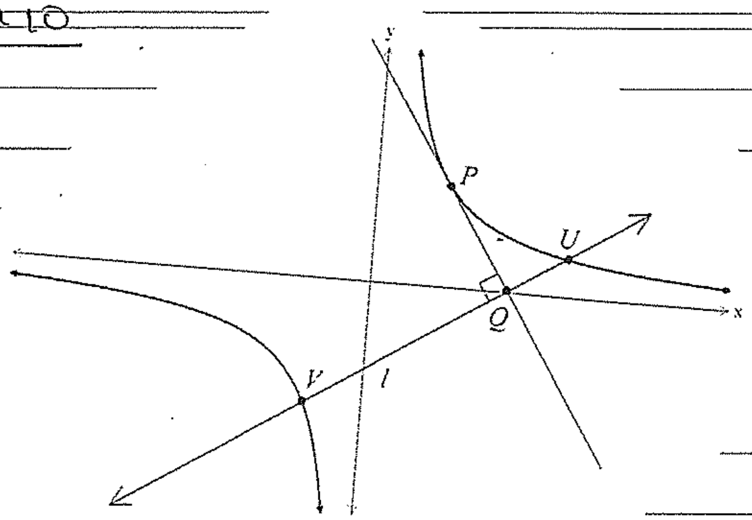
$$\therefore PF = \frac{b^2 e}{a} \times \frac{1}{e} = \frac{b^2}{a}$$

$$AF = h \Rightarrow ae - a = h = a(e-1)$$

$$\therefore PF = \frac{b^2}{a} = \frac{a^2(e^2-1)}{a} = \frac{a^2(e-1)(e+1)}{a}$$

$$= a(e-1)(e+1) = h(e+1)$$

Question 10



$$xy = c^2$$

(i) Differentiating both sides give,

$$xy' + y = 0 \quad \therefore \ y' = \frac{-y}{x}$$

$\therefore$ gradient at $P = -\dfrac{\frac{c}{t}}{ct} = \dfrac{-1}{t^2}$

$\therefore$ equation of the tangent:

$$y - \frac{c}{t} = \frac{-1}{t^2}(x - ct)$$

$$\therefore t^2 y - ct = -x + ct$$

$$\therefore x + t^2 y = 2ct \qquad (2)$$

(ii) coordinates of $Q \Rightarrow$ sub-in $y = 0$

$$\therefore x = 2ct$$

$$\therefore Q = (2ct, 0) \qquad (1)$$

(iii) gradient of the normal at $P = t^2$

$\therefore$ equation of l: $y - 0 = t^2(x - 2ct)$

$$y = t^2 x - 2ct^3 \qquad (1)$$

To find the coordinates of v,

(iv) $y = t^2x - 2ct^3$ — ①

$xy = c^2$ — ②

Solve simultaneously ① and ②

sub. in ② in ①

$$\frac{c^2}{x} = t^2x - 2ct^3$$

$$\therefore c^2 = t^2x^2 - 2ct^3x$$

$$t^2x^2 - 2ct^3x - c^2 = 0.$$

$$\text{mid point of } UV = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{sum of roots} = x_1+x_2 = -\frac{b}{a}$$

$$= \frac{2ct^3}{t^2} = 2ct$$

$$\therefore \text{mid point } m = \left(\frac{2ct}{2}, y\right) = (ct, y)$$

To find y, sub. in $x = ct$ in ①

$$y = t^2 ct - 2ct^3 = -ct^3$$

$$\therefore m = (ct, -ct^3)$$

3

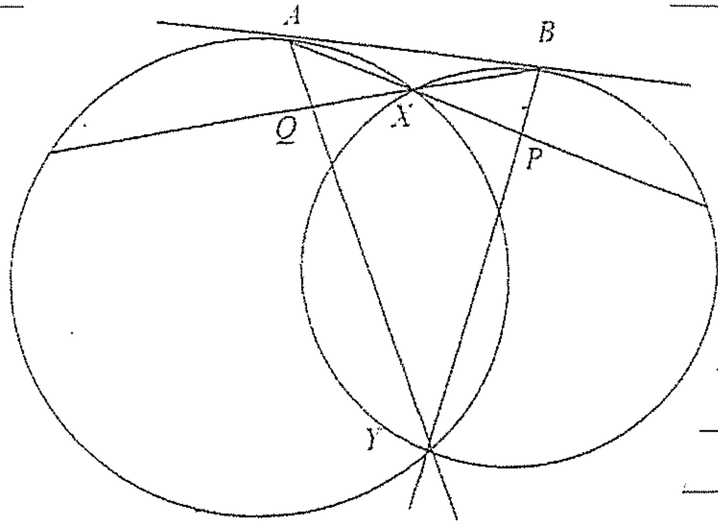
v) $X = ct$ $\therefore t = \frac{X}{c}$

$$Y = -ct^3$$

$$= -c \cdot \frac{x^3}{c^3} = -\frac{x^3}{c^2}$$

2

Question 10



(i) Join XY. Let $\angle BAX = x$, $\angle ABX = y$

$\angle BXP = x+y$ (In $\triangle BAX$, exterior angle = sum of interior opposite $\angle$'s)

$\angle BAX = \angle AYX$ (angle between tangent and chord = $\angle$ in alternate segment)
$= x$

similarly,
$\angle ABX = \angle BYX = y$

$\therefore \angle QYP = x+y = \angle BXP$

$\therefore$ XPYQ is a cyclic quadrilateral (exterior $\angle$ = interior opposite $\angle$).

4

(ii) Join PQ.

$\angle XPQ = \angle QYX = x$ ($\angle$ at circumference standing on the same arc as XPYQ is cyclic)

$\therefore \angle BAP = \angle XPQ = x$.

$\therefore AB \parallel QP$ (alternate angles equal)

3